CORRESPONDENCE.

Editor Analyst:

A copy of the ANALYST for July 1881 has just fallen into my hands in which I find "An Investigation of the Mathematical Relations of Zero and Infinity", by Professor Judson. In this article the author makes use of some equations, and the solution of the same, from Thomson and Quimby's Collegiate Algebra, as follows:—

"Messrs. Thomson and Quimby give the following illustration of a false interpretation of $a \div 0 = \infty$. (Algebra, Art. 348, p. 146.)

'Given $x^2 + xy = 10$ (1), and $xy + y^2 = 15$ (2), to find x and y.

Let $x = zy$. Then, from (1),

$$y^2 = \frac{10}{z^2 + z} \text{ (5), and from (2), } y^2 = \frac{15}{z+1} \text{ (6).}$$

From (5) and (6) $10z + 10 = 15z^2 + 15z$ (7); $\therefore z = \frac{2}{3}$ or -1.

Substituting -1 for z in (5) or (6) we have $y = \pm \infty$, $\therefore x = \mp \infty$; hence $\infty^2 - \infty^2 = 10$ and $\infty^2 - \infty^2 = 15$'.

That these results are incorrect is manifest; for if we eliminate y between (1) and (2) we have an equation of the second degree, which should have two roots only. But if $\pm \infty$ be roots, then an equation of the second deg. may have four roots. Adding (1) and (2), $x^2 + 2xy + y^2 = 25$, $\therefore x + y = \pm 5$. By substituting in (1), $\pm 5x = 10$, or $x = \pm 2$. Substituting in (2), $\pm 5y = 15$, $\therefore y = \pm 3$; and these are the only roots.

The correct interpretation of this example is, since when $z = -1$, $y^2 = 15 \div 0$, $\therefore z = -1$ is an impossible value for (1) and (2)."

In reply to Professor Judson, I will say:—

1. The elimination of y between (1) and (2) gives an equation of the 4th degree which must have four roots.

2. Hence it follows that $x = \pm 2$ and $y = \pm 3$ are not the only roots but $x = \pm \infty$ and $y = \mp \infty$ are also roots.

3. I have never before heard that $y^2 = 15 \div 0$ indicated impossibility except the impossibility of expressing the result in terms of a finite unit. If Prof. Judson will refer to the Algebra above named he will find that *zero* is defined as *an infinitesimal*, and the book does not recognize a zero which means nothing. The Professor is certainly wrong when he says that infinity cannot be a root of simultaneous equations. The equations above are each the equation of an hyperbola, and the two hyperbolas have common points at $x = \pm 2$, $y = \pm 3$, and also have a common asymptote, and therefore have infinity as roots.

E. T. QUIMBY.

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Editor Analyst:

The problem on p. 17 of the ANALYST of January 1882, "On the Computation of the Eccentric Anomaly", etc., reminds me of the method of solving numerical equations which, some years ago (Sept. 1876), I contributed to the ANALYST, and of which nobody seems to have taken notice.

This method, giving an acceleration of the third order, applies also to transcendental equations that have no multiple roots. I beg to subjoin its application to the above mentioned problem.

If $f(x) = 0$ be the equation to be solved, and $f'(x)$, $f''(x)$ denote the first and second differential quotient; if further the assumed initial value of the root be x_0, then is the corrected value

$$x_1 = x_0 - \frac{f(x_0)f'(x_0)}{f'^2(x_0) - \frac{1}{2}f(x_0)f''(x_0)}.$$

Applied to our problem, we have:

$$f(E) = E - e\sin E - m,$$
$$f'(E) = 1 - e\cos E,$$
$$f''(E) = e\sin E.$$

Now as $e = 0.2056$, by mere inspection we see that $E = 150°$ is a good initial value; for $\sin 150° = \sin 30° = \frac{1}{2}$, and the term $e\sin E = \frac{1}{2}(0.2056) = 0.1028$; the length of the arc of $1° = (\pi \div 180) = 0.0174533$, so that 0.1028 translated into degrees will give an arc of between $5°$ and $6°$; as $m = 143°$, the value $150°$ will give a small number for $f(E)$. The computation is now as follows:—

$$f(150°) = 150° - 5°.89003 - 143° = 1°.10996, \text{ in degrees of arc,}$$
$$= 0.01937 \text{ in length of arc.}$$
$$f'(150°) = 1 + 0.2056 \times 0.86603 = 1.178056;$$
$$f''(150°) = \tfrac{1}{2}(0.2056) = 0.1028.$$

$$\text{Now } E_1 = 150° - \frac{1°.10996 \times 1.178056}{(1.178056)^2 - \frac{1}{2}(0.01937 \times 0.1028)}$$
$$= 150° - 1°.10996 \times 0.84946 = 149°.05714$$
$$= 149°.\ 3'\ 25''.7$$

This value substituted in f will make $f(E_1) = -0.000009$
$$= 1''.85 \text{ in arc.}$$

It seems to me that this method is recommendable, especially in those cases where the several differential quotients admit of simple expressions.

DR. H. EGGERS.

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Editor Analyst:

Since the publication of the article on the Solution of Equations in the last ANALYST, I think I have discovered "the weak link in the chain" of its logic, as applied to the solution of the equation of the 5th degree.

It lies in the statement, near the bottom of page 5, that in the supplemental equations "one letter may be interchanged with another without disturbing any relations". If, in brief, numerical values be assigned to the letters, an interchange in the order of their arrangement materially disturbs the relations.

Thus if $x = \sqrt[5]{a^5} + \sqrt[5]{b^5} + \sqrt[5]{c^5} + \sqrt[5]{d^5} = 1+3+5+7$, the arrangement of $x = 1 + 3 + 7 + 5$ furnishes, when the substitution is made, a *different* equation from the one first given. As these letters are capable of twenty-four permutations, the result is the production of twenty-four general eq'ns of the 5th degree. But as only one is given for solution, an additional element is unfolded in the eq'n of the 5th degree, namely, a dependence of its solution, not only upon the value of the four letters a, b, c and d, but also upon the *order* of their arrangement. As these letters cannot be thus permuted and satisfy the given equation, it would appear that instead of one final equation being produced by elimination, "whichever three of the four unknown quantities are eliminated", four final equations would be produced, one each in a, b, c and d. It remains to determine by algebraic analysis, if possible, the character of the roots of these final equations, their mutual relations, the effect of permissable permutations, and whether they can be separated into groups, so as to permit of the reduction of the equations to the 4th degree; or, on the other hand, to determine, by algebraic methods within the comprehension of the ordinary algebraist, the impossi'ty of such reduct.

I regret that pressure of professional labors renders it impossible for me to further prosecute the work at present; but I shall be pleased if, per chance, I shall have contributed my mite towards awakening a renewed interest in the subject, trusting that further research will yet result in the development of a perfect and complete Theory of Equations.

T. S. E. DIXON.

Chicago, Ill., Feb. 13, 1882.

SOLUTION OF 393 BY R. J. ADCOCK.—"Two particles of masses m and m' respectively, are connected by a string passing through a small fixed ring and are held so that the string is horizontal; their distances from the ring being a and a', they are let go. If ρ and ρ' be the initial radii of curvature of their paths, prove that

$$\frac{m}{\rho} = \frac{m'}{\rho'}, \text{ and } \frac{1}{\rho} + \frac{1}{\rho'} = \frac{1}{a} + \frac{1}{a'}."$$